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ELASTIC WAVES AND ULTRASONIC NONDESTRUCTIVE EVALUATION

*Proceedings of the IUTAM Symposium on
Elastic Wave Propagation and Ultrasonic Evaluation
University of Colorado, Boulder, Colorado, U.S.A.
July 30 - August 3, 1989*

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WAVE PROPAGATION IN COMPOSITE MEDIA AND MATERIAL CHARACTERIZATION

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Characteristics of wave propagation in an undamaged composite medium are influenced by many factors, the most important of which are: microstructure, constituent properties, interfaces, residual stress fields, and ply lay-ups. Measurements of wave velocities, attenuation, and dispersion provide a powerful tool for nondestructive evaluation of these properties. In this paper we review recent developments in modeling of ultrasonic wave propagation in fiber and particle reinforced composite media. Additionally, we discuss some modeling studies of the effects of interfaces and layering on attenuation and dispersion. These studies indicate possible ways of characterizing material properties by ultrasonic means.

1. INTRODUCTION

Ultrasonic waves provide an efficient means of characterizing the effective mechanical properties of a nonhomogeneous material. Several theoretical studies show that for long wavelengths one can model the effective wave speeds and attenuation of plane longitudinal and shear waves propagating through a medium containing a distribution of inclusions or fibers. It is possible also to model the changes in speeds and attenuation of waves propagating in the presence of microcracks or voids. At long wavelengths the wave speeds predicted by these models are non-dispersive and hence provide the values for the static elastic moduli of the bulk material.

Speeds of propagation of elastic waves in the presence of a distribution of inclusions have been modeled [1-9]. References to other works can be found in those cited and in the review articles [10,11]. Although most of the early works dealt with spherical inclusions, effect of inclusion shape and orientation has been modeled in [4,6-9]. Anisotropic wave speeds caused by oriented ellipsoidal inclusions were modeled and compared with experiments [9].

As a special case of propagation through a particle-reinforced composite one can also obtain the results for a medium permeated by cracks or voids (pores). There are numerous works that have dealt with the problem of speed and attenuation of waves in such a medium. References to these

*The work reported here was supported in part by grants from the National Science Foundation (#INT-8521422 and #INT-8610487), by a grant from the Office of Naval Research (#N00014-86-K-0280; Scientific Officer: Dr. Y. Rajapakse), and by NASA (#NAGW-1388).

†Supported by a grant from the Natural Science and Engineering Research Council of Canada (#OGP007988).

can be found in [12-15]. Effect of void shape on phase velocities has been reported in [10] and [15].

Most of the works dealing with particle reinforced composites have assumed perfect bonding between the inclusions and the matrix material. Effect of interface (interphase) layers has received some attention [3,16-20]. It has been found that diffuse interface has very weak influence on wave propagation characteristics. However, effect of an interface layer is quite pronounced. Such a layer increases attenuation. Both the density and stiffness of the layer, and the curvature of the interface affect the attenuation.

The literature on wave propagation in a fiber-reinforced composite material is vast. There are numerous models that have been proposed to predict dispersion and attenuation of elastic waves propagating perpendicular to the aligned continuous fibers. References to many of the works can be found in [21]. Most of the reported works deal with isotropic fibers. Anisotropic fibers were considered in [21,22]. In [22] model predictions in conjunction with experiment were used to inversely determine the graphite fiber transverse isotropic elastic constants.

Because most structural composites are laminated or layered, wave propagation in laminated composite plates and shells has received considerable attention. Several papers in this volume deal with this problem and the reader is referred to those for additional references.

In this paper we will focus our attention on guided wave propagation in a composite plate. Two problems will be considered: (1) Guided waves in a cross-ply periodically laminated plate, and (2) dispersion of waves in a bonded plate with isotropic homogeneous layers. These two problems are chosen to illustrate the effect of layering and the interface (bond) layer properties on the dispersion characteristics of guided waves.

2. PROBLEM FORMULATION AND SOLUTION

In this section we will first consider a periodically laminated composite plate where each lamina is made up of a continuous fiber reinforced material. Then in the second part of the section we will discuss guided waves in a plate made up of two isotropic homogeneous layers bonded together by a thin layer of bond material.

2.1 Guided waves in a cross-ply laminated plate

Consider a cross-ply laminated plate, which is composed of alternate layers of continuous fiber reinforced materials of equal thickness. It will be assumed that fibers are oriented at 90° to one another in adjacent layers and that the configuration is symmetric in the plate. Thus the top and bottom layers have fibers oriented in the same direction. A global Cartesian coordinate system with origin on the mid-plane of the middle layer will be chosen. x -axis is chosen parallel to the direction of propagating guided waves which is either parallel or perpendicular to the fibers in the middle layers. y -axis lies in the middle plane and z -axis perpendicular to the plane. The thickness of the plate is taken to be H so that the thickness of each lamina is $h=H/n$, where n is the number of laminae.

If the wavelengths of the propagating waves are much larger than the fiber diameters and spacings then, as has been shown before [21,22], each lamina can be modeled as transversely isotropic with the symmetry axis parallel to the fibers. Thus the problem reduces to that of wave propagation in a plate with layers of transversely isotropic material, where the axes of symmetry in adjacent layers are perpendicular to one another. Our object here is to analyze the effect of the number of layers on dispersion of guided waves propagating either along the x -axis or along the y -axis.

Since we will be considering a large and varying number of layers it will be convenient to resort to a numerical technique in which the number and properties of layers can be altered arbitrarily without substantially changing the solution procedure. Such a technique was proposed earlier by

us [23] and also by others [24-26]. In [24-26] authors present a stiffness method in which the thickness variations of the displacements are approximated by quadratic functions of the thickness variable. The generalized coordinates in this representation are the displacements at the top, middle, and bottom of each layer. In [23] an alternative higher order polynomial representation was proposed where generalized coordinates were the displacements and *tractions* at the top and bottom of each layer. This was found to give better results at high frequencies. However, because both displacements and tractions were involved, it entailed much more cumbersome algebra than the scheme used in [24-26]. To avoid this algebraic complexity we will use the quadratic interpolation functions used in [24-26].

Since we consider waves propagating either in the symmetry direction or perpendicular to it in each layer, the problem separates into two uncoupled ones: plane strain in which the displacement components are u_x , 0, u_z , and SH or antiplane strain when the only non-zero displacement is u_y . In this paper we will consider the plane strain problem only.

In order to achieve numerical accuracy each lamina is divided into several sublayers. A local coordinate system $(x^{(k)}, 0, z^{(k)})$ is chosen in each sublayer with the origin in the mid-plane. The strain-displacement relations in each sublayer are, for non-vanishing strain components,

$$\epsilon_{xx}^{(k)} = u_{x,x}^{(k)}, \quad \epsilon_{zz}^{(k)} = u_{z,z}^{(k)}, \quad \epsilon_{xz}^{(k)} = \frac{1}{2} \gamma_{xz}^{(k)} = \frac{1}{2} (u_{x,z}^{(k)} + u_{z,x}^{(k)}) \quad (1)$$

where comma denotes differentiation. The stress-strain relation in this sublayer is

$$\{\sigma\} = [c^{(k)}] \{\epsilon\} \quad (2)$$

where

$$\{\sigma\}^T = [\sigma_{xx}, \sigma_{zz}, \sigma_{xz}] \quad (3a)$$

$$\{\epsilon\}^T = [\epsilon_{xx}, \epsilon_{zz}, \gamma_{xz}] \quad (3b)$$

$$[c^{(k)}] = \begin{bmatrix} c_{11}^{(k)} & c_{13}^{(k)} & 0 \\ c_{13}^{(k)} & c_{33}^{(k)} & 0 \\ 0 & 0 & c_{55}^{(k)} \end{bmatrix} \quad (3c)$$

For convenience the superscript (k) on u , σ , and ϵ has been dropped above and in the subsequent development. Using the interpolation polynomials in the z -direction, the displacement components are approximated as,

$$\{U\} = [N] \{q\} \quad (4)$$

where

$$\{U\}^T = [u_x, u_z] \quad (5a)$$

$$\{q\}^T = \{u_x^b, u_z^b, u_x^m, u_z^m, u_x^f, u_z^f\} \quad (5b)$$

$$[N] = \begin{bmatrix} n_1 & 0 & n_2 & 0 & n_3 & 0 \\ 0 & n_1 & 0 & n_2 & 0 & n_3 \end{bmatrix} \quad (5c)$$

In equation (5) the generalized displacements $u_x^b(x,t)$, $u_z^b(x,t)$, $u_x^m(x,t)$, $u_z^m(x,t)$, $u_x^f(x,t)$, and $u_z^f(x,t)$ are taken at the back, middle, and front (top) nodal surfaces of the sublayer. The interpolation polynomials n_i are quadratic functions given by

$$n_1 = -2z + 2z^2, \quad n_2 = 1 - 4z^2, \quad n_3 = 2z^2 + z \quad (6)$$

where $z = z^{(k)}/h^{(k)}$, $h^{(k)}$ being the thickness of the sublayer.

Using Hamilton's principle the governing equation for the entire plate is found [26] to be

$$[K_1]\{Q\}'' + [K_2]^*\{Q\}' - [K_3]\{Q\} - [M]\{\ddot{Q}\} = 0 \quad (7)$$

Here $[K_1]$, $[K_3]$, and $[M]$ are symmetric and $[K_2]^*$ is skew symmetric. Primes and dots denote differentiation with respect to x and t , respectively. $\{Q\}$ is the vector of all the nodal displacement components. We will consider propagating waves in the x -direction. Thus $\{Q\}$ is assumed of the form

$$\{Q\} = \{Q_0\}e^{i(kx - \omega t)} \quad (8)$$

Substituting (8) in (7) we get the eigenvalue problem

$$[-K_1k^2 + K_2^*ik - K_3 + M\omega^2]\{Q_0\} = 0 \quad (9)$$

Equation (3) can be solved to find ω for a given k or to find k for given ω . Some results are discussed in section 3.

2.2 Rayleigh-Lamb Waves in a Bonded Plate

The characterization of bond quality using ultrasonic techniques has long been a subject of study by many researchers. Ultrasonic methods provide a powerful tool for detecting debonding or weakening of bond strength. Their success in measuring bond strength largely depends on the understanding of the nature of the changes in the wave propagation characteristics due to the changes in the material properties of the bond layer.

A study of guided waves in thin layers can be found in [27]. For thin bonded layers there have been several studies [28-34]. In most of these the case of normal incidence is considered. Oblique incidence has been considered in [35]. In these studies the bond layer is approximated as a massless spring or a fluid layer that allows jump in the displacement keeping tractions continuous. Attempts at using this spring or slip model to detect weak bonding has shown its applicability to a class of bonds, although there are indications that adjustable coefficients may be needed to explain the behavior at high frequencies [35-37]. In [20,36,37] a shell model that combines the effects of density (inertia) and stiffness has been developed to study scattering from inclusions with thin interface (bond) layers. It is found that the effect of density dominates in the cases considered. Baik and Thompson [38] have also considered a shell model to analyze dispersive behavior and a density model has been used by Nayfeh and Nassar [39].

In an effort to assess the feasibility of characterizing bond properties by ultrasonic means we [40] have made a parametric study of the exact spring and density models for the thin interface layer in a bonded plate. A summary of the results is presented in the following.

We consider a sandwich plate of two outer layers and an interface thin bond layer of materials that are isotropic and homogeneous. A global Cartesian coordinate system (x, y, z) with origin at the free surface of the top layer, and x, y -axis parallel and z -axis perpendicular to the surface, is considered. For convenience our attention will be focused on the two-dimensional problem. Thus it is assumed that the displacement components $(u_x, 0, u_z)$ uncouples from the antiplane strain motion $(0, u_y, 0)$. Here we present some results for the plane strain case.

In plane strain deformation the nonzero displacements and tractions at an interface $z = \text{constant}$ form a four vector

$$\{S\} = [u_x, u_z, \tau_{xz}, \tau_{zz}]^T \quad (10)$$

Let $\{S^-\}$ and $\{S^+\}$ denote the values of the four vector $\{S\}$ as $z \rightarrow h^-$ and h^+ , respectively. Here $z = h$ is the depth of an interface from the top free surface. If the bond is perfect at the interface then

$$\{S^-\} = \{S^+\} \quad (11)$$

Now consider two layers occupying $0 \leq z \leq h_1$ and $h_2 \leq z \leq h_3$ are bonded together by a thin layer of thickness $h_2 - h_1 = h_0$. If h_0 is very small compared to h_1 , $h_3 - h_2$, and the wavelength of the propagating wave, then the thin layer is often approximated as one of vanishing thickness and the interface conditions are applied at $z = h_1 = h_2$. In the shell model it is assumed that

$$(S^+ - S^-) = \frac{1}{2} \begin{bmatrix} 0 & F \\ D & 0 \end{bmatrix} (S^- + S^+) \quad (12)$$

where

$$[F] = \text{Diag} \left[a \frac{h_0}{\mu_0} \quad a \frac{h_0}{\lambda_0 + 2\mu_0} \right] \quad (13)$$

$$[D] = \text{Diag} [-b\rho_0\omega^2 h_0 \quad -b\rho_0\omega^2 h_0] \quad (14)$$

where a, b are some parameters, and λ_0 , μ_0 , and ρ_0 are the Lamé constants and density, respectively, of the layer. In writing (12) a harmonic time dependence of the form $e^{-i\omega t}$ has been assumed, ω being the circular frequency. Note that when $a=0$ one obtains the density model and for $b=0$ the spring model holds.

Equation (12) is often an adequate approximation for the thin layer, the regimes of validity of which have not been systematically investigated. In [40] a parametric study has been performed for a typical plate. Some of these results are discussed in the next section.

3. NUMERICAL RESULTS AND DISCUSSION

In this section we present some selected numerical results showing the effect of the number of layers and bond properties on the dispersion of guided waves.

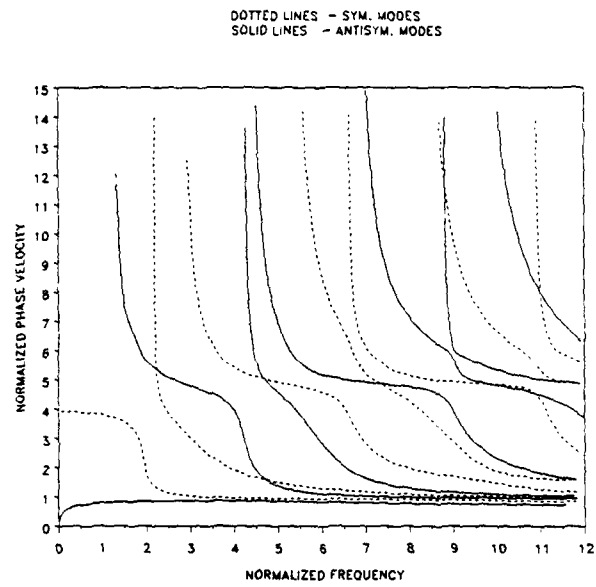
3.1 Dispersion of Waves in a Laminated Plate

In order to understand the effect of the number of laminae we consider a graphite fiber-reinforced laminate. The properties of 0° and 90° laminae are given in Table 1. Here 0° signifies fibers aligned with the wave propagation direction (x -axis) and 90° signifies fibers aligned with the y -axis.

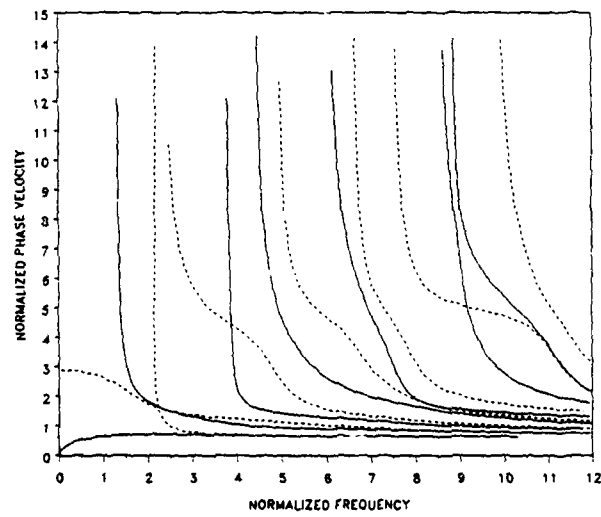
Table 1. Properties of the laminae
(All stiffnesses are in units of 10^9 N/m²)

Layer	ρ (g/cm ³)	c_{11}	c_{33}	c_{13}	c_{44}	c_{55}
0° lamina	1.2	1.6073	0.1392	0.0644	0.0350	0.0707
90° lamina	1.2	0.1391	0.1392	0.0350	0.0707	0.0350

Figures 1(a) and 2(a) show the variation of phase velocities of different modes with frequency in three-layered ($0^\circ/90^\circ/0^\circ$) and 39-layered ($\dots 0^\circ/90^\circ/0^\circ \dots$) plates. Corresponding results for propagation in differently oriented plates ($90^\circ/0^\circ/90^\circ; \dots 90^\circ/0^\circ/90^\circ \dots$) are presented in Figures 1(b) and 2(b). It is seen that dispersion characteristics depicted by Figs. 1(a) and 1(b) are vastly different. However, Figs. 2(a) and 2(b) are remarkably similar. In fact, these results agree quite well with predictions of an effective modulus theory (for details the reader is referred to [41]). Thus it appears that for a sufficiently large number of layers the plate behaves isotropically in its plane. This is a remarkable result and clearly the number of layers necessary to show in-plane isotropic behavior must depend on the material properties and stacking sequence of the laminae. We hope to pursue this further in the future.

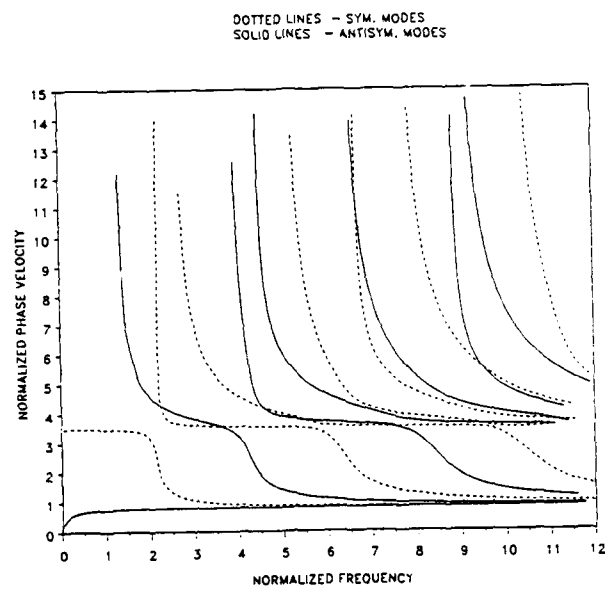


(a)

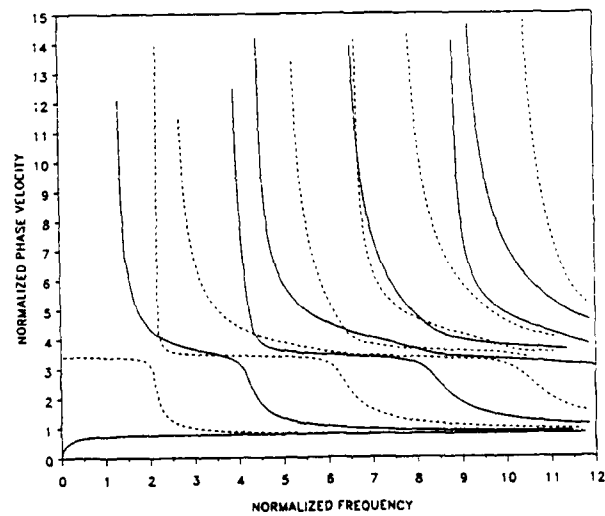


(b)

FIGURE 1
 Guided waves in a ($0^\circ/90^\circ/0^\circ$) three-layered plate.
 (a) Propagation in the 0° direction. (b) Propagation in the 90° direction.



(a)



(b)

FIGURE 2
Same as Figure 1 for a 39-layered plate.

3.2 Interface Layer Effects on Dispersion

To exhibit the implications of different approximations of thin interface layers on the dispersion of guided waves we consider a plate made up of an outer layer of gold on a nickel-iron substrate. Properties are given in Table 2. Properties of the interface layer are varied.

Table 2. Properties of a Bonded Plate
(Longitudinal wave speed= c_p , Shear wave speed= c_s)

Material	ρ (g/cm ³)	c_p (mm/ μ s)	c_s (mm/ μ s)	Thickness (mm)
Gold (au)	19.32	3.24	1.22	0.45
Bond	--	--	--	0.5
Fe - 42% Ni	8.10	4.86	2.60	10.0

Figures 3 and 4 show the dispersion curves predicted by the actual model and those obtained by the approximate spring and mass models. In these figures M is the ratio of the shear moduli of the bond layer and the gold layer, and D is the ratio of the densities. It is seen from Fig. 3 that when the bond layer has both high modulus and high density then the spring model predictions are higher than the actual. In this case the density model predictions are very close to the actual for all the modes except the third, when the results agree only at low frequencies. The reason for this anomalous behavior is not clear. Figure 4 shows the results for the case when the bond layer is very weak (low density and low modulus). It is seen that in this case the spring model agrees very well with the exact, whereas the density model predictions are higher. It is interesting to note that in both cases the two model predictions become close to the exact at high frequencies for all the modes except the first and the third. Other features (not shown) of interest are:

- both the spring and density model predictions agree with the exact when the bond is high modulus and low density;
- none of the models is very good for all the modes when the bond is low modulus and high density.

4. CONCLUSION

It has been demonstrated that ultrasonic velocity techniques provide sensitive means of characterizing the properties of bulk composite materials and interfaces between the different phases. It is also shown that dispersion characteristics of waves in a plate are very sensitive to the following parameters:

- Number of layers and stacking sequence in a laminated plate.
- Properties of the interface bond layer.

Thus the ultrasonic waves may be used to characterize these properties. Furthermore, it appears that a laminated plate with a sufficiently large number of layers can be modeled as effectively quasi-isotropic. This feature may have very important significance for ultrasonic characterization of thick composites.

ACKNOWLEDGEMENTS

The authors are grateful to Dr. P.-C. Xu for carrying out the parametric study of a bonded plate.

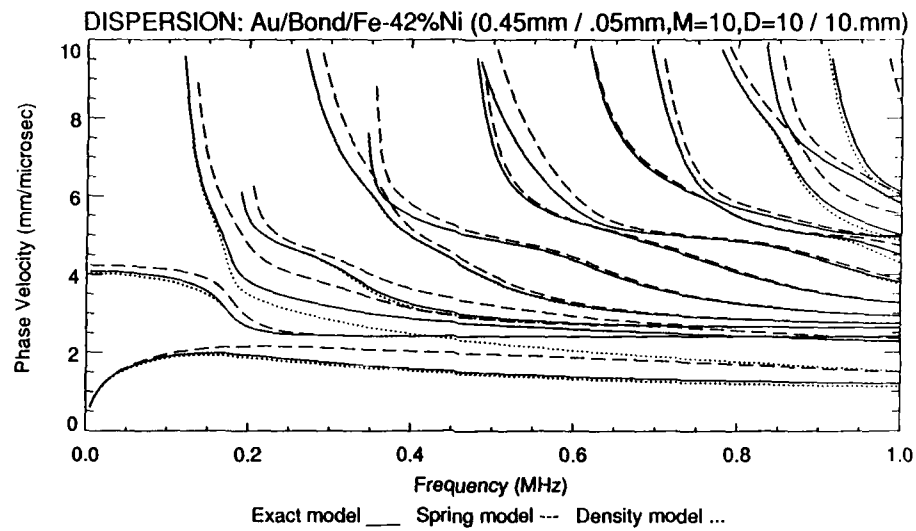


FIGURE 3
Effect of a heavy stiff bond layer on guided waves.

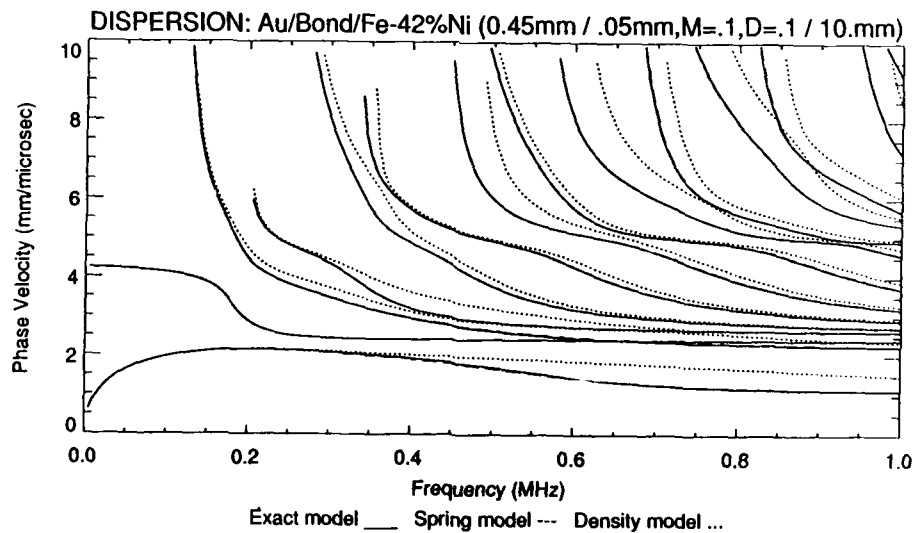


FIGURE 4
Effect of a light soft bond layer on guided waves.

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